Team Control Number

11576

Problem Chosen

B

2021

HiMCM/MidMCM
Summary Sheet

The Recognition, Prediction and Countermeasure for the Drought in Lake Mead

Lake Mead is the largest reservoir by volume in the nation and it supplies water for more than 25 million people in three Western States. However, the water level in Lake Mead declines continuously in recent years and has led to the water shortage in the contiguous states. The purpose of this report is to analyze and quantify the relationship between the inflow, outflow and loss of Lake Mead, and identify the criteria for drought periods based on water level. We are also expected to provide some methods for forecasting the future water level and formulate corresponding wastewater recycling plans to prevent drought.

In problem 1, the correlations between inflow, outflow and loss of Lake Mead were estimated according to the data from Bureau of Reclamation, and the correlations showed that the main reasons of decreasing water level were the decrease of inflow and the increase of outflow, while the loss was nearly consistent. Further, the water budget balance of Lake Mead was quantified, where the approximate inflow was 9 maf; the approximate outflow was 9.6 maf; and approximate evapotranspiration and bank storage change was 0.6 maf. The results represents that the water level of Lake Mead would decline at a rate of 12 ft per year. Finally, based on the definite integral and quadratic fitting model, the area was defined as the quadratic function of elevation approximately, and the volume approximation was defined as the cubic function of elevation with the parameters estimated by the least square method based on the data about the elevation, area and volumn.

In problem 2, according to the Colorado River Interim Guidelines issued by the Bureau of Reclamation, the criteria of water level that divides drought and non-drought was set as 1115ft. According to the criteria, the most recent drought period had a relatively higher duration fluctuation level even though its average water level remained same compared to earlier periods. Besides, assuming that 2013-2021 period's pattern would continue, the average water levels forecasted by ARIMA in 2025, 2030, and 2050 were 1049.82ft, 1024.57ft and 927.77ft; assuming that 2005-2020 period's pattern would continue, the average water levels forecasted by Trend-Seasonal-ARIMA in 2025, 2030, and 2050 were 1055.50ft, 1035.59t and 1006.42ft. The resulting ARIMA presented a significant decreasing trend, while Trend-Seasonal-ARIMA showed a flatter decreasing trend, predicting that the water level would remain above 1000ft by 2050.

In problem 3, based on the literatures about the wastewater recycling system, the construction cost and efficiency of the three types of wastewater treatment facilities were determined at first. Then, based on the water level of Lake Mead in 2018 and 2030, and the capacity of wastewater treatment facilities in Nevada in 2018, the expected wastewater treatment capacity in 2030 would be 79700000 m^3. Besides, the number of wastewater treatment facilities was calculated by minimizing the construction cost respect to the expected wastewater treatment capacity in 2030, and the branch and bound method was designed to solve the model. The quantities of wastewater treatment facilities were 2, 8 and 9 and the total construction cost would be $602767700. Finally, we averaged the expenditures between 2021 and 2030, and obtained the final implementation plan for wastewater recycling facilities shown in Figure 11. In addition, we use sensitivity analysis to demonstrate the reliability of the model.

One of the contributions of this paper is that the relationship between the accurately water level prediction model and criteria for drought periods were established, which can better reflect the actual drought conditions and development trends. Besides, the wastewater recycling scheme based on the integer optimization model could also maximize uses the limited funds to solve the drought problem in Lake Mead.

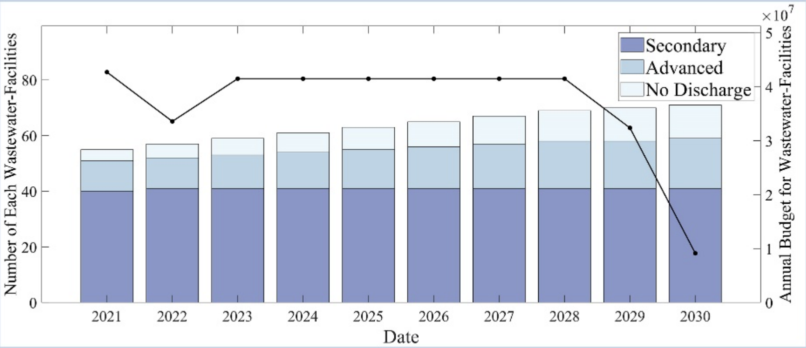
Keywords: Drought recognition, ARIMA, Trend-Seasonal-ARIMA, Integer optimization.

OUTSTANDING LAKE MEAD RESCUE PLAN

Lake Mead is the largest reservoir by volume in the nation and it supplies water for more than 25 million people in three Western States. However, the water level in Lake Mead declines continuously in recent years and has led to the water shortage in the contiguous states. The correlations showed that the main reasons of decreasing water level were the decrease of inflow and the increase of outflow, while the loss was nearly consistent. Further, the water budget balance of Lake Mead is shown in the Table, it represents that the water level would decline at a rate of 12 ft per year.

Lake Mead Water Budget Factord	Value
Approximate annual inflow into Lake Mead	9.0 maf
Approximate annual outflow from Lake Mead	-9.6 maf
Approximate annual Lake Mead evaporation loss	-0.6 maf
Water balance	-1.2 maf

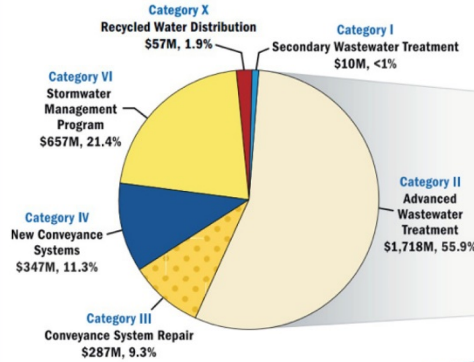
Furthermore, if the most recent drought period's pattern continues, the forecasting average water levels in 2025, 2030, and 2050 were 1049.82ft, 1024.57ft and 927.77ft. Therefore, in order to prevent the continuous drought, we suggest to build 2, 8 and 9 wastewater treatment facilities respectively. The annual budget will be less than 60 million dollars and obtained the final implementation plan for wastewater recycling facilities is shown in the Figure. The coordinate axis on the left corresponds to the quantity of waste water facilities, the three colors correspond to the types of facilities respectively, and the coordinate axis on the right corresponds to the budget invested in waste water facilities every year.



It was expected that the water level of Lake Mead would restore to 7.9 ft every year through the set of wastewater treatment facility to solve the drought crisis around Lake Mead.

How wastewater treatment facilities

1. **Screening;**
2. **Settling;**
3. **Digestion;**
4. **Dewatering;**
5. **Aeration**
6. **Export**



HIMCM 2021

Protect Lake Mead! Report by Team #11576

Contents

1 Introduction

1.1 Background

Lake Mead is the largest water reservoir inthe United States. On August 16, 2021, the Bureau of Reclamation announced the first-ever water shortage declaration on the Colorado River. As droughts around the world have been increasing, researchers continue to develop means torecycle water more efficiently. One method is to recycle wastewater that flows out of our sinks, toilets, and showers. You may think this is impossible, but thistechnology already exists and is improving. Under most current circumstances, this wastewater goesthrough several treatment processes prior to being released into local waterways. A treatment facility uses various techniques to meet fit-for-purpose pecifications. These specifications are the requirements necessary to bringwater in from a particular source, treat it to get it to the quality level needed for a particular purpose,and then make it available for that purpose. Some purposes include agricultural irrigation, domesticwater supply, industrial and commercial use, and recharging groundwater.

1.2 Requirements

- The amount of water is a function of inflow, outflow, and loss. Please identify and describe factors that impact inflow, outflow, and loss in Lake Mead. Discuss therelationship of these factors and their relative influences on the volume and water level ofLake Mead. Consider how youwould verify the elevation, area, and volume relationships, describe what information and data you would need and discuss how you would use mathematics tocalculate these measures.
- The two problem data files provide information about Lake Mead waterlevels given by elevation in feet above mean sea level. Briefly discuss overall patterns in the historical data for Lake Mead water levels. Define your criteria for drought periods and identify the beginnings andends of periods of drought. Comment on how the most recent drought period compares toearlier ones. Develop two models for the water level in Lake Mead as a function of the year based on two assumptions such as
  - Model 1: Consider data from only the most recent drought period and assume the mostrecent drought period's pattern continues.
  - Model 2: Use water level data from 2005 - 2020 and assume this period's patterncontinues.

  and predict the water level in Lake Mead in the years 2025, 2030, and 2050. Compareand evaluate your two models and their predictions.
- Address the impact on future waterusage demands and consider if the recycling of wastewater could make up all or a part of anyfuture shortfalls based on your models and water level predictions in problem 2, identify and describe the factors you would include in a plan to recycle wastewater. Considerthe decisions local leaders would need to make and the priorities they might set that wouldimpact your plan. Besides, Describe your plan and how you would measure the impact of implementing your plan.

1.3 Our Approach

The problem requires us to study the current water balance of Lake Mead, and predict the water level in the future based on its historical water level data, and finally give a set of wastewater recycling solutions under the premise of considering the actual local conditions.

- Based on the physical geographical conditions and related data of Lake Mead, we quantified the annual water budget balance sheet of Lake Mead. We gave the correlation between several different influencing factors and the elevation, area, and volume of Lake Mead.
- Based on the Lake Mead Management Guidelines issued by the Bureau of Reclamation, the standard for judging drought in Chengdu based on the water level of Lake Mead is defined, and the water level and drought development trends of 2025, 2030, and 2050 are carried out using the ARIMA model and Trend-Seaonal-ARIMA model respectively.
- We introduced the principles and characteristics of the current mainstream wastewater recycling system, and based on the current scale and status of wastewater recycling facilities in Nevada, established an integer programming model to solve the optimal wastewater treatment setup and construction plan for the period 2021-2030, and explained measure the impact of implementing our plan.

2 General Assumptions

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

- **Assumption 1**: Factors such as geography-related water sedimentation and microbial action are negligible relative to the water volume of Lake Mead itself.
  ↪ **Justification**: According to the water flow of the Colorado River Basin and the outflow supplying to the states, the majority of the water flow of Lake Mead is already included, so ignoring the factors which are still quantitatively controversial in academics will not affect our model.
- **Assumption 2**: The water level data of Lake Mead is predictable.
  ↪ **Justification**: Although the water level of Lake Mead is affected by many factors, its water level data series can often include this information. The observation points before and after the water level series are commonly not independent but based on some autocorrelation. Linked together, so the past water level series can be utilized to predict the future water level.
- **Assumption 3**: The main body of the wastewater recycling plan is to build large-scale wastewater treatment facilities rather than relying on residents' non-technical self-distribution.
  ↪ **Justification**: Many technically sophisicated procedures are required to recycle wastewater and finally discharge it back to Lake Mead which is too complex for the residents to implement by themselves and can only rely on extensive wastewater treatment facilities.

- **Assumption 4**: The current wastewater treatment technology we are considering will not be broken by further technological development in the future.
  $\hookrightarrow$ **Justification**: Technological breakthroughs are often unpredictable and irregular. Considering technological breakthroughs in the model will make the results unpredictable.

- **Assumption 5**: Assume that the data reviewed is accurate.
  $\hookrightarrow$ **Justification**: Our data sources are mainly attachments and related academic papers. We can establish a more credible quantitative model based on the fact that these data are not falsified.

3 The Development of Models

3.1 Notations

Important notations used in this article are listed in Table 1.

Table 1: Notations

Symbol	Description	Unit
$V(t)$	The volumn of Lake Mead in tth periods	maf
$L(t)$	The waterlevel of Lake Mead in tth periods	feet
$F_i(t)$	The ith water budget balance factor of Lake Mead in tth periods	maf
x_1, x_2, x_3	The number of three type wastewater facilities	-

3.2 Model Overview

On August 16, 2021, the Bureau of Reclamation announced the first water shortage statement historically, which directly affected the water supply balance in the surrounding area and led to a series of social impacts. In summary, models that this article needs to establish are

- **Water Budget Balance**: Firstly, we establish a statistical analysis model to quantify the balance of water inflow and outflow of Lake Mead, and the correlation between the factors that affect the balance;

- **Calculation Relationship**: Secondly, we establish the integral and fitting function describes the relationship between the elevation, area, and volume of Lake Mead, and explains the data structure required for the solution;

- **Drought Recognition**: Thirdly, we establish a drought recognition model to identify the problems faced by Lake Mead through the historical water level data of Lake Mead Drought conditions, and analyze the characteristics of different drought periods from the three perspectives of duration, average water level, and fluctuation;

- **Waterlevel Forecast**: Fourly, we establish the ARIMA model and the Trend-Seaonal-ARIMA model were established to predict the water level of Lake Mead in the future based on different assumptions;

- **Wastewater Recycling Plan**: Finally, based on the cost and efficiency of the existing wastewater treatment system, we establish an integer programming model to plan the construction of different types of wastewater treatment plants included in the wastewater recycling system of Lake Mead in the future.

In summary, the whole modeling process is shown in Figure 1

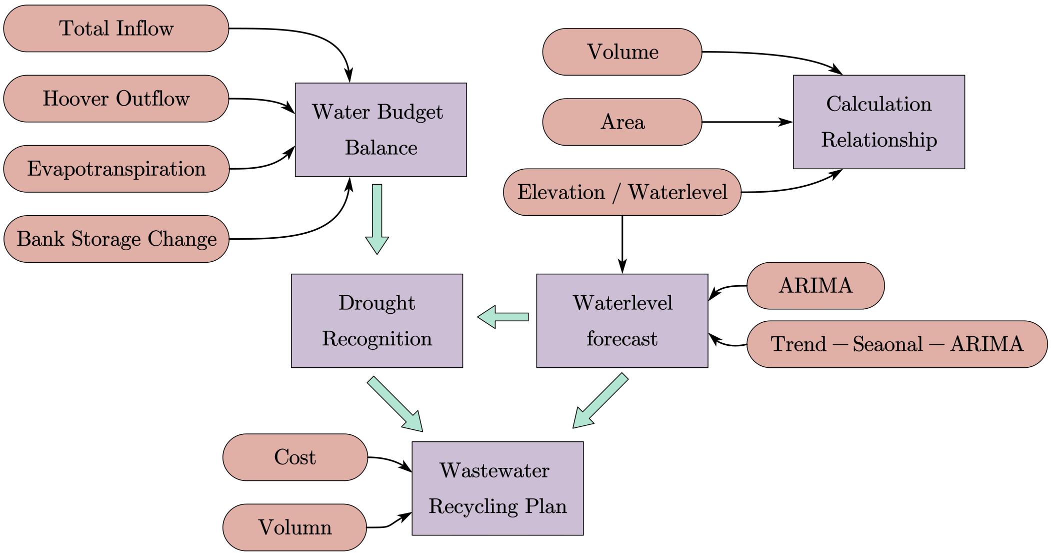


Figure 1: Model Overview

4 Water Budget and Geographical Situation of Lake Mead

4.1 Analysis

Requirement 1 asks for the analysis of the factors affecting Lake Mead's water resources budget balance and their relationship based on its geographical and hydrological information, and the construction a model that can describe the elevation, area, and volume of Lake Mead.

For the first question, three features need to be clarified

- Impact of the geographical location and natural conditions of Lake Mead on its water resources
- Composition of inflow and outflow of water of Lake Mead
- Quantification of the inflow and outflow of water of differents parts of Lake Mead

For the second question, two features need to be clarified:

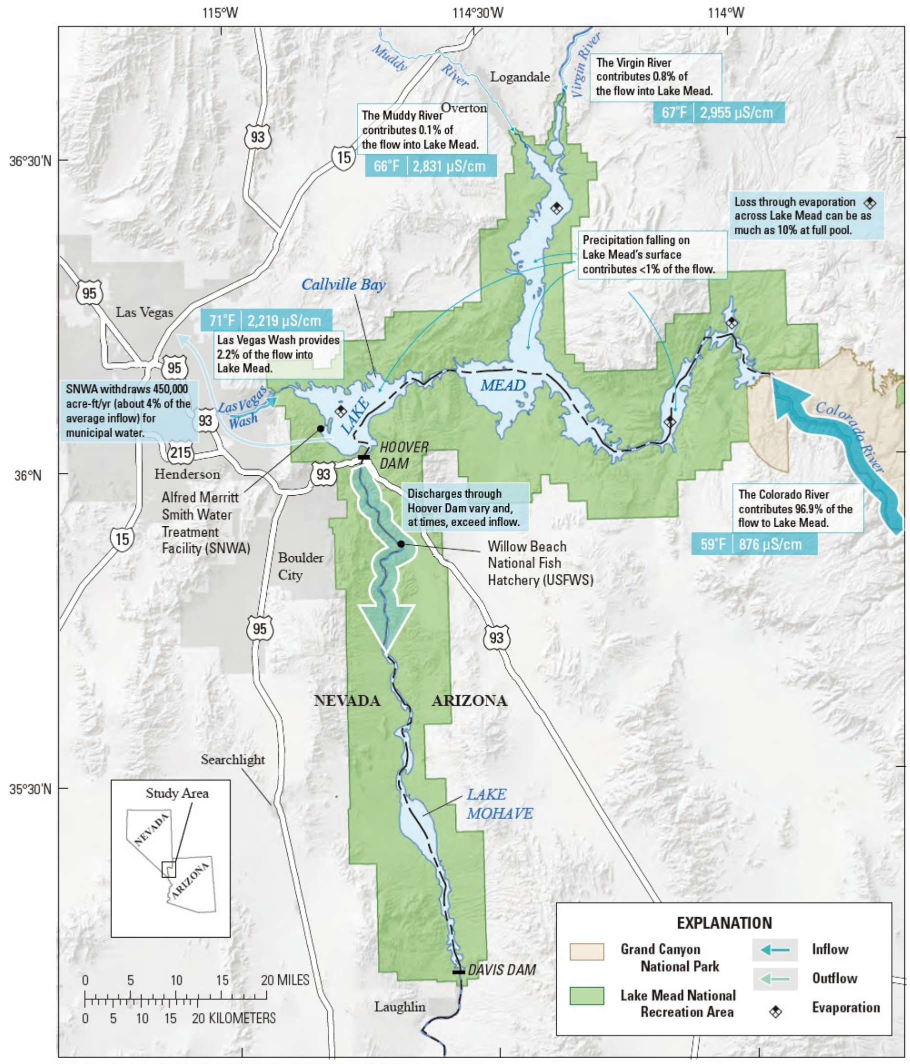
- The suitable function to fit the relationship between the elevation and area for the calculation utilizing the Newton-Leibniz formula.
- The required data structure to solve the model mentioned above.

To summrize, based the situation mentioned above, this section will investigate the above two problems in Section 4.2 and 4.3.

4.2 Lake Mead Water Budget Balance

4.2.1 Lake Mead and the Colorado River Basin

Lake Mead is a reservoir formed by the Hoover Dam on the Colorado River in the southwest of the United States on September 30, 1935 (see Figure 2). It is located between Nevada and Arizona, 39 kilometers east of Las Vegas. Ranking by the water volume, it is the largest reservoir in the United States. Today, Lake Mead provides water resources for nearly 20 million people and parts of the farmland in Arizona, California, Nevada and parts of New Mexico.



Base from National Park Service and U.S. Geological Survey digital data,1:675,000
Universal Transverse Mercator projection, Zone 11
North American Datum 1983

Figure 2: Colorado River Basin Water Flow

The right to use the water resources of the Colorado River is allocated to seven states in the basin according to a series of treaties. Under the constraints of the treaties, the river is divided into the upper basin and the lower basin in order to be managed by authorities: the upper basin includes part of Colorado, New Mexico, Utah and Wyoming; The lower basin includes part of Arizona, California and Nevada. Therefore, the model of the water budget balance of Lake Mead should take into account the relevant information of the Colorado River Basin and other states located near the the Colorado River Basin. The following parts in this section will analyze and quantify the water resource budget balance of Lake Mead from three features: inflow, outflow and loss of water.

4.2.2 Lake Mead Water Budget Balance

According to Healy et al. [2007], the Earth's water resources exists on its surface in the form of oceans, ice, lakes, rivers, streams and wetlands; It also exists underground in the form of soil water and groundwater, or in the atmosphere in the form of vapor. Water circulates continuously in different forms and undergo different paths. In some paths, water circulates rapidly; For instance, rain falling from the atmosphere into corn fields in summer may evaporate back into the atmosphere in a few hours or a few days. However, travel time in other paths is measured in years, decades, centuries or more - for example, glaciers contain water that fell from the atmosphere thousands of years ago.

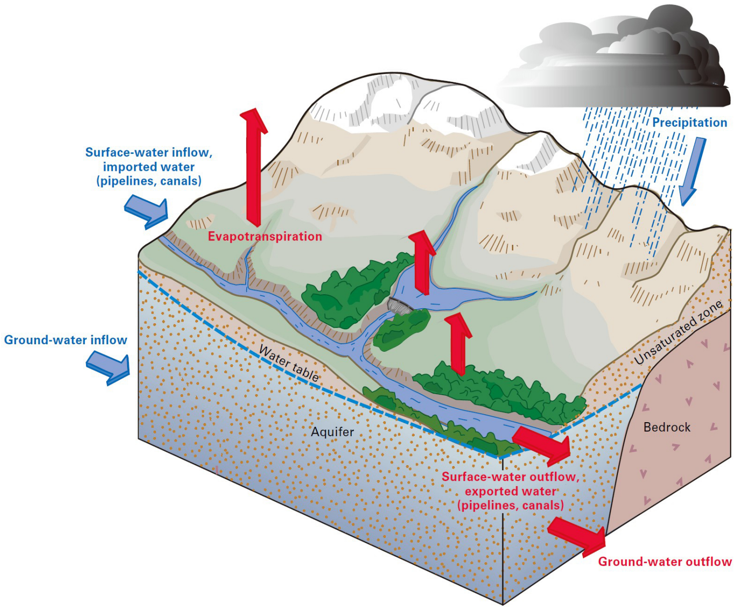


Figure 3: Natural water cycle

Considering the past research of biodiversity protection, we evaluate a project's utility through economic factor, ecological factor, and feasibility factor as mentioned in the analysis above. In the case of Lake Mead, the equation of water balance of year t can be expressed as

$$V_{\text{total}}(t) = V_{total}(t-1) + V_{\text{input}}(t) - V_{\text{output}}(t) - V_{\text{loss}}(t), \tag{1}$$

where,

- $V_{\text{total}}(t)$ represents the total amount of water of Lake Mead in the year of t.
- $V_{\text{input}}(t)$ represents the total inflow of water of Lake Mead in the year of t.
- $V_{\text{output}}(t)$ represents the total outflow of water of Lake Mead in the year of t.
- $V_{\text{loss}}(t)$ represents the total loss of water of Lake Mead in the year of t.

Next, we will analyze and quantify the composition of total inflow of water, total outflow of water and total loss of water respectively based on equation (1).

- **Total Inflow of water**:
  Firstly, the main water supply source of the reservoir is the river. Lake Mead is mainly supplied by the Colorado River. Through a series of compacts and treaty obligations, the flow finally allocated to Lake Mead is about 26.1 maf. In addition, according to Barnett and Pierce [2009], we selected the Diamond Creek measuring station as the observation point of inflow water, and obtained the water flow trend data of Mead Lake in recent years. In addition, there is a small part of wastewater and precipitation from treated wastewater transferred back to Lake Mead, TRMM (special precipitation satellite) provides precipitation data near Lake Mead.

- **Total Outflow of water**:
  Secondly, the total outflow of water, according to Bureau of Reclamation, the water output of Mead lake mainly includes about $3.67 \times 10^8 m^3$ supplied to Nevada and about $5.5 \times 10^8 m^3$ supplied to Las Vegas every year. There is also a small allocation of other parts, such as the river bank storage.

- **Total Loss of water**:
  The Mojave desert around Lake Mead is one of the highest deserts in the United States (Singh et al. [2016]). Therefore, we have collected data from the website of Bureau of Reclamation and calculated that the annual evaporation loss of Lake Mead is estimated to be about 0.6 maf, almost 7% of its average annual inflow.

According to our analysis above, we have sorted out the factors influencing Lake Mead's water budget into the following four parts,

- F_1 : Total Inflow
- F_2 : Hoover Outflow
- F_3 : Bank storage change
- F_4 : Evapotranspiration

Let f_{it} be the data numbered t from factor i, we can calculate the correlation coefficient between factor i and factor j as

$$r_{ij} = \frac{\sum_{t=1}^{n} \left(f_{it} - \bar{f}_i\right)\left(f_{jt} - \bar{f}_j\right)}{\sqrt{\sum_{t=1}^{n} \left(f_{it} - \bar{f}_i\right)^2} \sqrt{\sum_{t=1}^{n} \left(f_{jt} - \bar{f}_j\right)^2}}, \tag{2}$$

where $\bar{f}_i = \sum_{t=1}^{n} f_{it}$ represents the average of factor i, and the the correlation coefficient matrix is calculated in MATLAB is shown in Figure 4. It can be seen that there is an obvious negative correlation between bank storage change and total inflow, while there is an obvious positive correlation between the bank storage change and the Hoover outflow. The reason for this phenomenon is that bank storage tends to be saturated under sufficient inflow with small change. When inflow is insufficient, it tends to make up for inflow with a large change. In addition, there is no obvious correlation between other variables. When total inflow and Hoover outflow are large, which means water flows frequently, the value of evapotranspiration is also large.

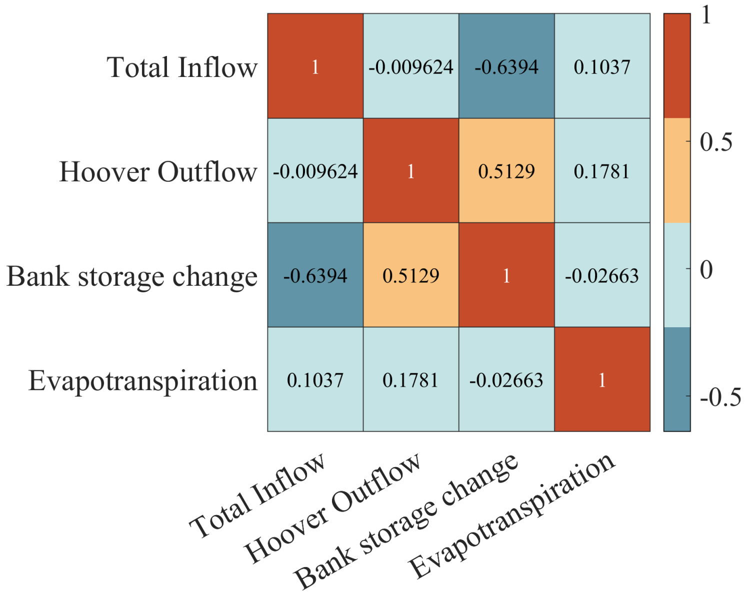


Figure 4: Correlation of each factors

To sum up, based on the data obtained from what's mentioned above, we can conclude with the overall water budget balance in Table 2. From Table 2 we know that the overall annual water budget of Lake Mead has operated with a deficit of 1.2maf in recent years, which means that if this value continues, the water level of Lake Mead will decline at a rate of 12ft per year.

Table 2: Lake Mead Water Budget

Lake Mead Water Budget Factord	Value
Approximate annual inflow into Lake Mead	9.0 maf
Approximate annual outflow from Lake Mead	-9.6 maf
Approximate annual Lake Mead evaporation loss	-0.6 maf
Water balance	-1.2 maf

4.3 The Relationships of Elevation, Area, and Volume

Considering the irregular shape of Lake Mead, the cross-sectional area $A(L)$ corresponding to different elevations L is not the same, but there is a certain correlation. Therefore, it can be considered to use integration to calculate the volume of general irregular objects $V(L)$,

$$V(L) = \int_{L_0}^{L} A(l)\mathrm{d}l, \tag{3}$$

where l_0 represents the elevation of the lake and l is the integral variable. Then, by observing equation (3), it is found that the key to calculating the volume is to identify the functional relationship between the elevation and the corresponding cross-sectional area, and infer from the data. Then we assume that the cross-sectional area is a quadratic function of elevation

$$a(L) = Al^2 + BL + C, \tag{4}$$

where a, b, c are the parameters to be estimated. By bringing equation (4) into equation (3), the relationship between volume and elevation is

$$V = \frac{a}{3}(L^3 - L_0^3) + \frac{b}{2}(L^2 - L_0^2) + c(L - L_0), \tag{5}$$

where, $l_0$is the parameter to be estimated. As shown in the equation (5), four parameters needs to be estimated. Therefore, the above parameters can be estimated by the least square method based on a large number of corresponding data including different elevations, surface areas and volumes of Lake Mead.

$$\begin{aligned} \hat{a}, \hat{b}, \hat{c} &= \arg\min \sum_{i=1}^{n} \left[A(L) - A^*\right]^2, \\ \hat{L}_0 &= \arg\min \sum_{i=1}^{n} \left[V(L, \hat{a}, \hat{b}, \hat{c}) - V^*\right]^2. \end{aligned} \tag{6}$$

Only considering the data provided in Table 1 of the issue, the corresponding fitting could be obtained by calculation in MATLAB, as shown in Figure 5. It showed that when there were only four groups of data, the above model could obtain accurate fitting results. If more data is given, equation (6) can still be utilized to bring in the data to fit the parameters to obtain a more accurate relationship between the elevation, area and volume of Lake Mead.

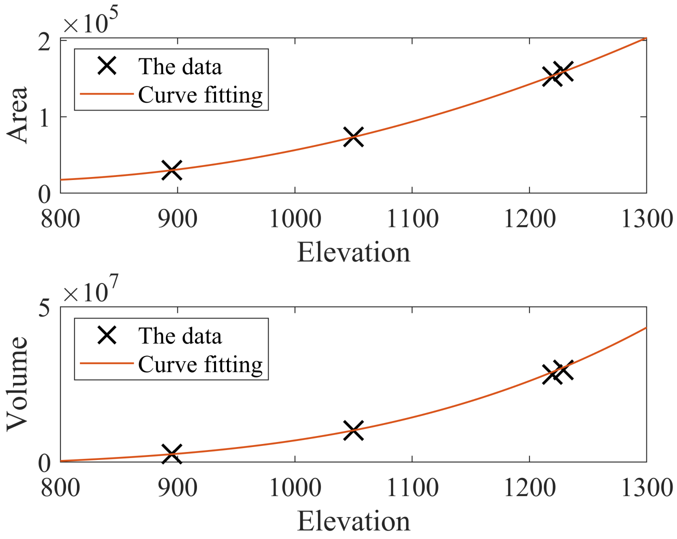


Figure 5: Elevation, area, and volume curve fitting

5 Drought Mode and Water-Level Forecast of Lake Mead

5.1 Analysis

Requirement 2 requests us to discuss and predict the changing trend of water level at Lake Mead based on historical data in the attachment. Then, based on certain definitions of drought, we are required to identify droughts and compare their characteristics that took place at Lake Mead during different time periods. Therefore, this section would independently investigate three issues in 5.2 and 5.3.

- From the perspectives of duration and fluctuation, define drought period based on data provided in the attachment.
- Predict future water level at Lake Mead with ARIMA model
- Separate water level data into three parts including trend term, seasonal term and irregular term. Then, predict future water level at Lake Mead according to these terms.

5.2 Drought Recognition based on Water Level Change Trend

According to of Reclamation [2007], Lake Mead Reservoir is allowed to adjust water level in accordance with actual water consumption, which aims for relieving drought pressure for surrounding areas. The water level at Lake Mead is therefore a significant factor indicating drought patterns in the vicinity. From Section 4.2, Lake Mead usually offers 7.5maf of water each year to the demand side. As long as the elevation is lower than 1115ft, the amount of water remained in Lake Mead is incapable of supporting such demand. This results in shortage of water in the lower basin, and it further influences water usage in Nevada and Arizona. The relationship between water level and drought is shown in Figure 6

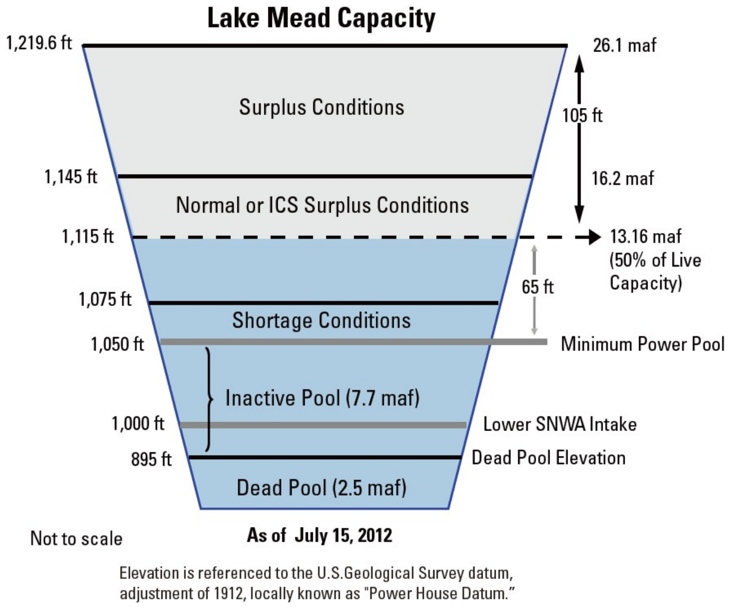


Figure 6: Drought degree quantification

Considering the actual situation, the criteria of water level that divides drought and non-drought is 1115ft. In other words, if the water level at Lake Mead exceeds this value, it is considered that no drought takes place nearby. A trend line is drawn using data from the attachment.

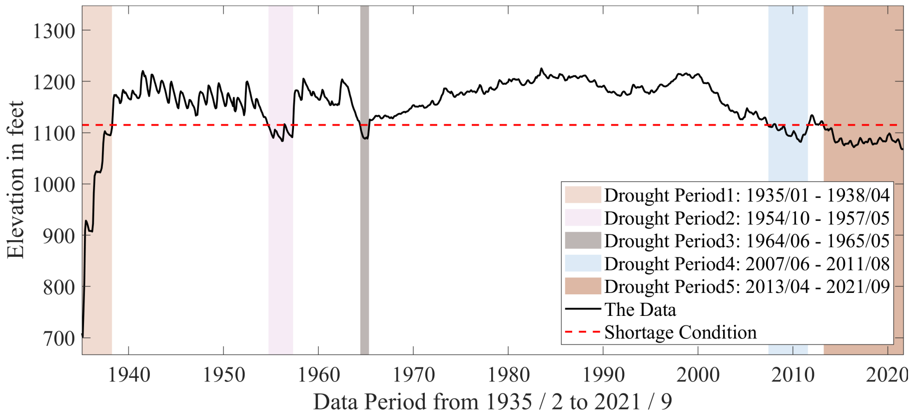


Figure 7: Water level trend and drought period recognition

As shown in Figure 7, Lake Mead has experienced five long drought periods. Even though the quantity of water in the reservoir is relatively low in the first drought period of Lake Mead, there has been a fast increase in its elevation. Then, the water level dropped below the drought level respectively in 1958 and 1966. The water level has been kept above the line until 2007, where Lake Mead underwent a relatively long time of drought. The most recent drought, started in 2013, was the longest, lasting for more than eight years. The water level at that time was a significant plummet from the historical peak. Additionally, we defined three indices to describe characteristics of drought periods

- **Duration**: the number of consecutive months $N_i,\ i = 1, 2, 3, 4, 5$ that the elevation at Lake Mead is below the drought level. Define L_{it} as the water level at Lake Mead on day t of the ith drought period.
- **Average elevation**: the average water level within drought period, calculated by:

$$\text{Average}_i = \frac{1}{N_i}\sum_{t=1}^{N_i} L_{it}, \tag{7}$$

- **Fluctuation level**: the variability of water level during drought periods, quantified by standard deviation and calculated by:

$$\text{Deviation}_i = \sqrt{\frac{1}{N_i}\sum_{t=1}^{n}\left(L_{it} - \text{Average}_i\right)^2}. \tag{8}$$

Based on the equations (7) and (8), we summarize the characteristics of the five drought periods into the Table 3.

Table 3: The characteristics of the five drought periods (unit: ft)

	Date range	Average	Duration	Fluctuation
Period1	1935-01 - 1938-04	986.02	39	110.84
Period2	1954-10 - 1957-05	1098.25	31	8.33
Period3	1964-06 - 1965-05	1095.76	12	8.61
Period4	2007-06 - 2011-08	1099.69	60	9.29
Period5	2013-04 - 2021-09	1085.18	102	9.90

The results show that the recent drought has a relatively higher duration fluctuation level even though its average elevation remains the same compared to other drought periods. The two characteristics that differentiate the recent droughts from the others further influenced the water supply and worsen the water shortage at Lake Mead. Combined with the water budget provided in Section 4.2, the elevation at Lake Mead is decreasing at a rate of 12ft/year. There should be urgent policies implemented to tackle such problem, and we will be discussing this issue in Section 6.

5.3 Waterlevel Forecast of Lake Mead

5.3.1 Waterlevel Forecast based on ARIMA model

In the section 5.1, we discussed the impact of drought assuming that the most recent drought period's pattern continues. This allows us to utilize recent data to predict future water elevation and drought. Under such circumstances, we choose to use the water level data in the most recent drought for prediction. Define the known water level array as $\{L_t\}_{t=1}^{n}$ so that the prediction can be modelled using autoregressive integrated moving average model (ARIMA), which is capable of processing non-stationary time series

$$\Delta L_t = a_0 + a_1 \Delta L_{t-1} + \varepsilon_t, \tag{9}$$

where, $\Delta L_t = L_t - L_{-1t}$ represents the first-order difference of the raw data; $\varepsilon_t \sim N(0, \sigma^2)$ is the residual of the model; and a_0, a_1 are parameters. Based on least squares method, the approximate value of parameters can be obtained

$$\hat{a_0}, \hat{a_1} = \arg\min \sum_{t=2}^{n} \left[\Delta L_t - a_0 - a_1 \Delta L_{t-1}\right]^2 . \tag{10}$$

We applied this in MATLAB and obtained the model fitting result shown in Figure 8.

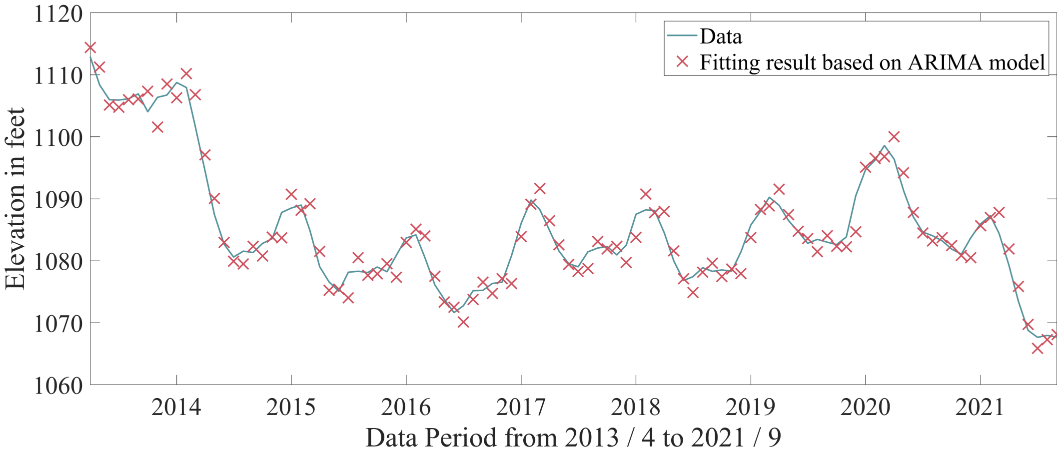


Figure 8: Data fitting effect based on ARIMA

Then, the water level forecast results for 2025, 2030 and 2050 based on ARIMA moedel is shown in Table 4

Table 4: Water level forecast results based on ARIMA (unit: ft)

	2025	2030	2050
JAN	1052.14	1026.89	930.09
FEB	1051.72	1026.47	929.67
MAR	1051.30	1026.04	929.25
APR	1050.88	1025.62	928.82
MAY	1050.45	1025.20	928.40
JUN	1050.03	1024.78	927.98
JUL	1049.61	1024.36	927.56
AUG	1049.19	1023.94	927.14
SEP	1048.77	1023.52	926.72
OCT	1048.35	1023.10	926.30
NOV	1047.93	1022.68	925.88
DEC	1047.51	1022.26	925.46
Mean	1049.82	1024.57	927.77

It can be seen that the average elevations at Lake Mead in 2025, 2030 and 2050, predicted by ARIMA model, are respectively 1049.82ft, 1024.57ft and 927.77ft, forming a decreasing trend.

5.3.2 Waterlevel Forecast based on Trend-Seaonal-ARIMA model

Compared to Section 5.3.1, we are not assuming the same drought conditions in this Section. Instead, we will be using data starting from 2005. For long term data, we divide the time series into trend term, seasonal term and irregular term, which can be expresssed by

$$L_t = T_t + S_t + I_t, \tag{11}$$

where,

- Trend term reflects the long term tendency of time series data. We use moving average method to calculate the trend terms in the data

  $$T_t = \frac{1}{n}\sum_{i=0}^{k} Y_{t-i} \tag{12}$$

  with $k = 12$ is the parameters of moving average method;

- Seasonal term reflects the periodic tendency of time series data. We use arithmetic average method to calculate the seasonal terms in the data

  $$S_t = \begin{cases} \frac{1}{45}\sum_{i=1}^{15}\left[Y_{(i-1)*3+3} + Y_{(i-1)*3+4} + Y_{(i-1)*3+5}\right], & \text{Spring} \\ \frac{1}{45}\sum_{i=1}^{15}\left[Y_{(i-1)*3+6} + Y_{(i-1)*3+7} + Y_{(i-1)*3+8}\right], & \text{Summer} \\ \frac{1}{45}\sum_{i=1}^{15}\left[Y_{(i-1)*3+9} + Y_{(i-1)*3+10} + Y_{(i-1)*3+11}\right], & \text{Autumn} \\ \frac{1}{45}\sum_{i=1}^{15}\left[Y_{(i-1)*3+12} + Y_{(i-1)*3+1} + Y_{(i-1)*3+2}\right], & \text{Winter} \end{cases}. \tag{13}$$

- Irregular terms represent the terms that include inconsistent information after excluding all trend terms and seasonal terms from the array. In other words, $I_t = L_t - T_t - S_t$. We will keep using the ARIMA model for the array $\{I_t\}_{t=1}^{n}$.

•

We applied this in MATLAB and obtained the model fitting result shown in Figure 9.

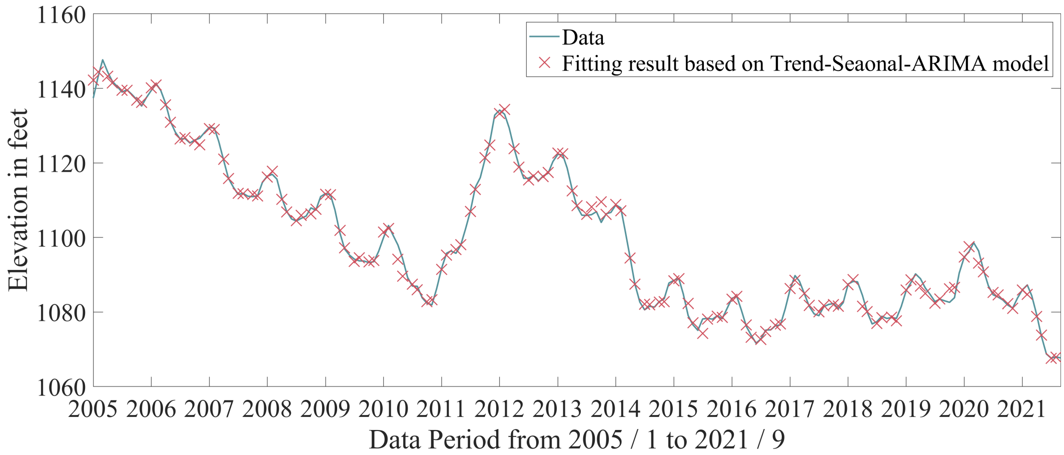


Figure 9: Data fitting effect based on Trend-Seaonal-ARIMA model

Then, the water level forecast results for 2025, 2030 and 2050 based on ARIMA moedel is shown in Table 5. It showed that both models produce a decreasing trend of Lake Mead's water level. However, the rate of change shown in the Trend-Seasonal-ARIMA model was flatter than the ARIMA model. The average elevations in 2025, 2030 and 2050, according to the Trend-Seasonal-ARIMA model, would be 1055.30ft, 1035.59ft and 1006.42ft.

Table 5: Water level forecast results based on Trend-Seaonal-ARIMA (unit: ft)

	2025	2030	2050
JAN	1062.36	1042.32	1013.14
FEB	1063.56	1043.49	1014.25
MAR	1062.01	1041.97	1012.56
APR	1058.33	1038.37	1008.99
MAY	1054.36	1034.47	1005.24
JUN	1051.62	1031.78	1002.69
JUL	1051.07	1031.24	1002.13
AUG	1051.91	1032.07	1002.98
SEP	1051.60	1031.77	1002.68
OCT	1051.94	1032.09	1003.03
NOV	1051.96	1032.11	1003.06
DEC	1055.30	1035.40	1006.23
Mean	1055.50	1035.59	1006.42

From the results, the first model presented a significant decreasing trend, predicting that the elevation would fall below 1000ft by 2050; the second model showed a flatter decreasing trend, predicting the water level to remain above 1000ft by 2050. According to the predictions, it is crucial to develop policies to reverse the trend of decreasing water level and addressing the issue of drought. We will be discussing this issue in the Section 6.

6 The Recycling Plan of Wastewater of Lake Mead

6.1 Analysis

Requirement 3 requires us to discuss if the wastewater recycling can relieve future water shortage efficiently according to the water level prediction model given in Section 5. Three features need to be clarified

- The relationship between the treatment efficiency and cost of current wastewater treatment system;
- Other factors that should be considered when implementing specific wastewater treatment plans;
- Comparison between the possible budget from local leaders and the estimated wastewater treatment cost;

Therefore, based on the situation above, this section will be researching on the features of wastewater treatment system in Section 6.2 and the implementing of recycling plan in Section 6.3.

6.2 The Features of Wastewater Treatment System

In Nevada, wastewater generally can be divided in to two categories

- Water utilized for watering plants, water injection or washing cars indoor;
- Water utilized for showering, laundry or flushing the toilet outdoors;

Figure 10: Wastewater recycling process

Among the two types of wastewaters, water utilized outdoors either evaporates, or flow along the streets and enters the water drains, then goes through the water reservoirs and finally goes into Las Vegas Wash. Whereas indoor water enters the wastewater treatment facilities through a series of pipes called collection system for treatment and finally discharged into Lake Mead. The Wastewater Treatment System was designed to ensure appropriate treatment of public wastewater for public health, the environment and the downstream water quality. The process of The Wastewater Treatment System is shown in Figure 10, it is mainly the following steps

- Large items in wastewater are removed by screening process, then the wastewater enters the settling tanks;
- Scums are cleared from the surface in the settling tanks;
- The grit and sludge are removed, then send the wastewater into dewatering facilities, while the remaining wastewater returned to the treatment process;
- Remaining wastewater is transferred into the aeration tanks where bacteria absorb the extra nutrients in the wastewater, such as phosphorus and organics in the wastewater;
- Apply a series of filtration and disinfection, and finally export the purified water to Lake Mead through Las Vegas Wash.

To ensure the system is applicable, the balance between the treatment efficiency and cost of the Wastewater Treat System is required. According to Hernández-Chover et al. [2018], we obtained the operation data of 217 wastewater treatment factories shown in Table 6, and the primary need shifted from secondary treatment to advanced treatment as facilities have been upgraded.

Table 6: The Features of Wastewater Treatment System

Type of Facilities	Volumn (unit: m^3)	Cost (unit: $)
Secondary	167573	1213900
Advanced	1316258	9130300
No Discharge	3985557	32387700

As shown in Table 6, volume refers to the water treatment rate of each waste water facilities, and cost refers to the funding required to build a waste water facilities. The higher the cost of waste water facilities, the greater the amount and efficiency of wastewater that can be treated.

Table 7: The Number of Facilities in Nevada

Type of Facilities	2004	2012	2018
Secondary	44	38	38
Advanced	3	9	10
No Discharge	4	3	3
Total	51	50	51

As the driest state in the US, Nevada has approximately 6775 miles of wastewater pipes connecting 50 different wastewater treatment facilities until 2018. Comparing to other states, many collection systems in Nevada are relatively new. According to EPA estimates, current wastewater treatment projects in Nevada requires about $3.08 billion. 55.9% of the budget are utilized for advanced wastewater treatment processes. Therefore, the priorities in the plan are

- The water level and drought crisis of Lake Mead;
- Annual budget deficit;
- Technical development of wastewater treatment plant

6.3 The Recycling Plan and Implementing of Lake Mead

According to the description of wastewater treatment system in Section 6.2, the number of three types of wastewater facilities was estimated to deal with the drought caused by the continuous decline of water level based on the water level data of Lake Mead and the previous number of wastewater facilities in Nevada. Therefore, according to the processing capacity and construction cost of the three types of 'waste water facilities' given in Section 6.2, define x_1 , x_2 , x_3 as the quantity of wastewater facilities to be established in 2030, then the overall wastewater treatment capacity is

$$\begin{aligned} \text{Volumn} &= 167573x_1 + 1316258x_2 + 3985557x_3, \\ \text{Cost} &= 12359x_1 + 91303x_2 + 323877x_3. \end{aligned} \tag{14}$$

Therefore, the planning of wastewater facilities can be transformed into the optimization of minimizing the construction cost under the constraint of the amount of wastewater. The constraint of the amount of wastewater, combined with the prediction of drought in 2030 in Section 5.3 and the existing wastewater facilities in Nevada given in Table 7, the expected amount of wastewater in 2030 can be calculated by

$$\frac{\text{Volumn}_{2018}}{\text{Volumn}_{2030}} = \frac{L_{2020} - 1219}{L_{2030} - 1219}. \tag{15}$$

That is, the amount of wastewater to be treated in 2030 is $7.97\ times 10^7 m^3$. Therefore, the integer optimization model of wastewater facilities is established as

$$\begin{aligned} &\min \text{Cost} = 1235900x_1 + 9130300x_2 + 32387700x_3, \\ &s.t. \quad \begin{cases} 167573x_1 + 1316258x_2 + 3985557x_3 \geq 7.97 \times 10^7, \\ x_1 \geq 38, \quad x_1 \geq 3, \quad x_1 \geq 10, \\ x_1, x_2, x_3 \in integer. \end{cases} \end{aligned} \tag{16}$$

We use the branch and bound algorithm method in MATLAB to solve the integer programming problem in equation (16). It was found that the number of secondary, advanced and no discharge facilities that should be built were 2, 8 and 9 respectively, and the estimated total investment in 10 years was US $602767700. Finally, if all wastewater facilities are built at one time, it will create a huge budget gap, which is not applicable considering the perspective of local leaders. Therefore, we averaged the corresponding budget into 2021-2030's plan. Figure 11 showed the number of waste water facilities and the corresponding investment curve. In Figure 11, the coordinate axis on the left corresponds to the quantity of waste water facilities, the three colors correspond to the types of facilities respectively, and the coordinate axis on the right corresponds to the budget invested in waste water facilities every year. In order to alleviate the financial pressure as much as possible, we have arranged the planned expenditure evenly over a period of 10 years. It was expected that the water level of Lake Mead would restore to 7.9 ft every year through the set of wastewater treatment facilities to solve the drought crisis around Lake Mead.

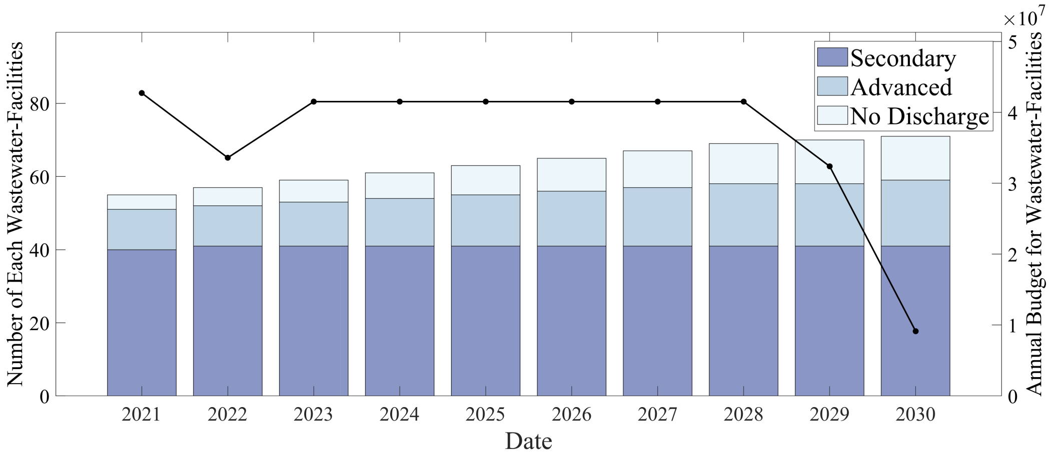
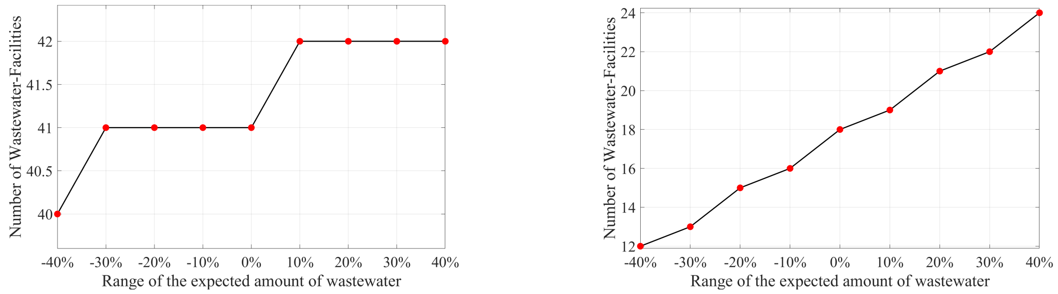


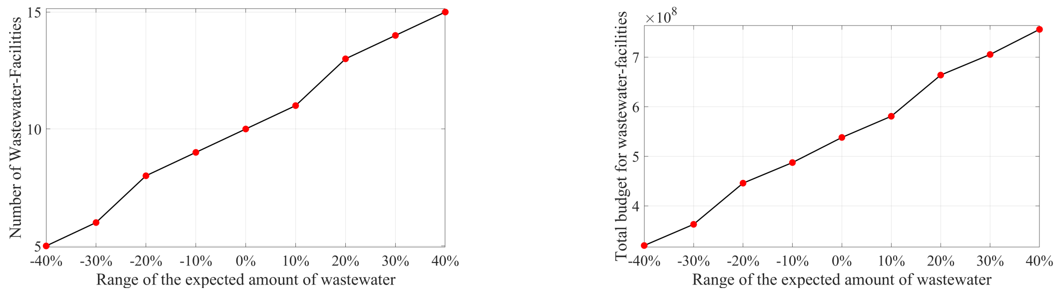
Figure 11: The Wastewater Recycling Plan in 2021-2030

6.4 Sensitivity Analysis

In Section 6.3, the exogenous factors are introduced to estimate the number of wastewater facilities and the corresponding budget: the expected amount of wastewater. Therefore, the relationship between results and the exogenous factors is approximated shown in the Figure 12.



(a) Secondary (b) Advanced



(c) No Discharge (d) Total budget

Figure 12: Sensitivity Analysis

It is indicated that the number of wastewater facilities and the corresponding budget increased with thethe expected amount of wastewater margin, which reflects the effectiveness of facilities on wastewater recycling. The trend of the model obtained by sensitivity test is consistent with the actual situation, which also proves the rationality and robustness of the model.

7 Model Evaluation

7.1 Advantages

- Based on the dynamic data of the various components of Lake Mead's water resources, we have obtained the data-based correlation between factors, which was more convincing;
- Our time series forecasting model based on the autocorrelation of historical water level data itself is objective;
- We obtained the optimal number of wastewater treatment facilities of different specifications through the integer programming model, and specified a set of reasonable and feasible wastewater recycling plans considering local financial pressure and conducted a sensitivity analysis to demonstrate the stability of the model.

7.2 Possible Improvements

- If we had more elaborate data, we could get a dynamic Lake Mead water resources budget balance sheet.
- Due to the limited time to complete, the wastewater recycling program we provided only involves the installation of large-scale wastewater treatment facilities, and did not consider possible technological progress. If there was enough time, a more comprehensive analysis could be established.

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Appendices

Appendix A Tools and software

Article written and generated via LaTeX, free distribution.
Graph generated via MATLAB R2020a and Xmind Zen, free trial license.
Calculation using MATLAB R2020a.

Appendix B The MATLAB Code

B.1 The Code for Problem 1

```
clc, clear, close all
Elevation = [1229 1219.6 1050 895]';
Area = [159866 152828 73615 30084]';
Volume = [29686054 28229730 10217399 2576395]';
p1 = polyfit(Elevation, Area, 2);
Ahat = @(p1, L)p1(1)*L.^2 + p1(2)*L + p1(3);
Vhat = @(p2, L)p1(1)/3*L.^3 + p1(2)/2*L.^2 + p1(3)*L + p2;
Err_AL = @(p1)mean((Ahat(p1, Elevation)-Area).^2./Area.^2);
Err_VL = @(p2)mean((Vhat(p2, Elevation)-Volume).^2./Volume.^2);
fminoptions = optimoptions('fmincon');
fminoptions.Display = 'iter';
fminoptions.ConstraintTolerance = 0;
fminoptions.FiniteDifferenceType = 'central';
fminoptions.MaxFunctionEvaluations = inf;
fminoptions.OptimalityTolerance = 0;
fminoptions.MaxIterations = inf;
p2 = fmincon(Err_VL, 0, [], [], [], [], [], [], [], fminoptions);
```

B.2 The Code for Problem 2

```
clc, clear, close all
load Problem
y1 = Elevation(Drought_Period{5});
d1 = Date(Drought_Period{5});
Mdl = arima(1, 1, 1);
EstMdl = estimate(Mdl, y1);
E1 = infer(EstMdl, y1);
numperiods = (2026 - 2022)*12 + 3;
Yhat1 = repmat(y1(end), numperiods+2, 1);
YMSE = zeros(numperiods+2, 1);
for i = 3:numperiods+2
    [Yhat1(i), YMSE(i)] = forecast(EstMdl, 1, Yhat1(i-2:i-1));
end
Yearhat = Date(81:431);
R2_model1 = 1 - var(E1(:))/var(y1(:));
```

B.3 The Code for PProblem 3

```
clc, clear, close all
load Problem
Vols = [167573 1316258 3985557];
Cost = [1213900 9130300 32387700];
x0 = [39; 10; 3];
y1 = mean(Elevation(Date>="2003-12-01" & Date<"2005-01-01"));
y2 = 1024.57;
V1 = Vols*x0;
V2 = V1*abs(y2-1219)/abs(y1-1219);
x1 = fmincon(@(x1)fun1(x1, Cost), [6; 2; 5], - Vols, - V2, [], [], x0);
x1 = round(x1);
function [finalCost, x1] = fun1(x1, Cost)
    x1 = round(x1);
    finalCost = Cost*x1(:);
end
```